Analysis of Indeterminate Contact Forces in Robotic Grasping and Contact Tasks

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Background

- Analysis of contact forces: necessary in various fields in robotics
  - Grasping
  - Manipulation
  - Fixturing
  - Walking

- Rigid-body + Coulomb friction
  - simple and widely used
  - Contact forces can be indeterminate
Coulomb friction

- A friction force must be within its friction cone
  - “Local” constraint on the feasibility

- Question: Is every combination of locally-feasible friction forces also feasible?

  Answer: No!
Combination of friction forces

(Example)

Combination of locally-feasible contact forces may be infeasible

(“Global” constraint on combinations of friction forces)
Constraint on combinations of friction forces

- A static friction force acts only in the opposite direction of the trend of virtual sliding.
- Only a part of combinations of virtual slidings are feasible due to contact kinematics.

Only a part of combinations of static friction forces are feasible.

OK

NG

- virtual sliding
- static friction force

[Omata and Nagata 00 T-RA]
[Omata 01 ICRA]
**Relationship between sliding and friction**

- **Virtual** sliding must be distinguished from **actual** sliding.

<table>
<thead>
<tr>
<th>Contacts in actual sliding</th>
<th>Contacts not in actual sliding</th>
</tr>
</thead>
<tbody>
<tr>
<td>in virtual sliding</td>
<td>not in virtual sliding</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Kinetic friction</th>
<th>Static friction</th>
<th>No friction</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Actual sliding is prevented by static friction)</td>
<td>(No need to prevent sliding by static friction)</td>
<td></td>
</tr>
</tbody>
</table>
We can find feasible friction forces by investigating feasible \( \dot{\dot{Y}} \).
Unreasonable results in Omata’s formulation (1)

- Adding a contact point makes a grasp infeasible
Unreasonable results in Omata’s formulation (2)
Objective

- To modify Omata’s formulation to exclude such unreasonable results
  - We present a procedure to calculate the set of possible indeterminate contact forces
  - We also present a technique for computation reduction
Why such unreasonable results?

- Problem in Omata’s formulation:
  Constraint on actual instantaneous sliding is applied to virtual sliding

\[
\begin{bmatrix} W^T & J \end{bmatrix} \begin{bmatrix} V \\ -\dot{\theta} \end{bmatrix} = TY
\]

- Some valid virtual slidings are excluded
- Thus some feasible friction forces are also excluded

We need a more relaxed constraint on virtual sliding
A new relaxed constraint (1)

- Constraint on virtual sliding of each *subset* of contacts

\[
B \begin{bmatrix} W^T & J \end{bmatrix} \begin{bmatrix} V \\ -\dot{\theta} \end{bmatrix} = TY
\]

\[
B := \text{diag}(b_1 I_3, \ldots, b_M I_3) \in \mathbb{R}^{3M \times 3M} : \text{selection matrix}
\]

\[
b_i = \begin{cases} 
1 & \text{when } P_i \text{ is in virtual sliding,} \\
0 & \text{otherwise.}
\end{cases}
\]

When \( B = I \): Identical solution to Omata’s formulation

Otherwise: Additional solutions may be found
A new relaxed constraint (2)

\[ B \begin{bmatrix} W^T & J \end{bmatrix} \begin{bmatrix} V \\ -\dot{\theta} \end{bmatrix} = TY \]

- Only selected contact points are constrained by contact kinematics
- Unselected contact points cannot generate static friction forces
- Consider \textbf{every} subset of contacts to obtain total set of contact forces
Example: Application of new constraint (1)
Example: Application of new constraint (2)
Procedure to calculate the set of indeterminate contact forces

- Set of possible indeterminate contact forces for each subcase: solution of linear inequalities

\[
\begin{align*}
    f &= Ck, \quad k \geq 0 & \text{Coulomb friction} \\
    A^T f &= w & \text{equilibrium equation} \\
    T^T (I_{3M} - B) f &= 0 & \text{unselected contacts cannot generate friction forces} \\
    ST^T f &\leq 0 & \text{friction forces act only to prevent virtual slidings}
\end{align*}
\]

- Total set of possible indeterminate contact forces: \textit{union} of the above solutions
Problem in new formulation

- Calculation for every subset of contact points
  - Combinatorial computation
  - Exponential complexity w.r.t. the number of contact points

→ Computation reduction is desired
A sufficient condition to skip unnecessary subsets

- Fingertip link has only one contact point and a matrix $Z$ is nonsingular.
- In this case, we can safely ignore the subsets in which the contact point is unselected.
  - Proof: see proceedings.
  - $Z$ is usually nonsingular.
Example: Computation reduction

- finger 1
- finger 2

- object

- selected contact
Special cases where computation reduction is inapplicable

Cases where $Z$ is singular:

- Contact point motion caused by outermost joint is in contact tangent space
Summary

- Omata’s formulation on indeterminate contact forces generates unreasonable results in some cases
- A modified formulation to exclude such unreasonable results is proposed
  - Procedure to calculate the set of possible indeterminate contact forces
  - Technique for computation reduction

Future work:
- Application to various problems of robotic grasping and manipulation [Maeda 05 ICRA] [Maeda 06 ICRA] [Makita 07 ISHF]
- Further computation reduction